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Radiation from Rectangular Waveguide with Ferrite Slabs

The problem of radiation from a rectangular waveguide completely filled with transversely magnetized ferrite has been discussed by Tyras and Held [1]. The fundamental TE mode in a rectangular waveguide containing two transversely magnetized symmetrical slabs placed against the side walls has been discussed by Lax and Button [2]–[4], and numerous references to this subject have been made by several other authors [5]–[7]. In the present communication we consider the radiation from the open end of a rectangular waveguide with two ferrite slabs of different thickness placed against the side walls, magnetized by two different static transverse magnetic fields, as shown in Fig. 1. Assuming harmonic time variation $e^{j\omega t}$, the electric fields for the fundamental TE mode are given by

$$E_y = A \sin k_1\left(\frac{a}{2}+x\right) e^{-j\gamma z} \quad \text{Region I}$$

$$E_y = B \sin k_2\left(\frac{a}{2}-x\right) e^{-j\gamma z} \quad \text{Region II}$$

$$E_y = \left[C \sin k_3\left(\frac{a}{2}+x\right) + D \cos k_3\left(\frac{a}{2}+x\right)\right] \cdot e^{-j\gamma z} \quad \text{Region III} \quad (1)$$

where γ is the propagation constant and

$$k_1^2 = \omega^2\epsilon_1\mu_{e1} - \gamma^2 \quad \text{Region I}$$

$$k_2^2 = \omega^2\epsilon_2\mu_{e2} - \gamma^2 \quad \text{Region II}$$

$$k_3^2 = \omega^2\epsilon_0\mu_0 - \gamma^2 = k^2 - \gamma^2 \quad \text{Region III} \quad (2)$$

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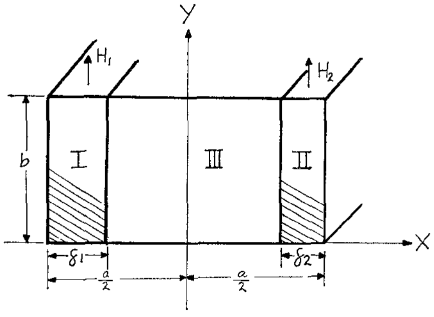

Fig. 1. Waveguide aperture.

where

$$\mu_{e1} = \frac{\mu_1^2 - K_1^2}{\mu_1}; \qquad \mu_{e2} = \frac{\mu_2^2 - K_2^2}{\mu_2}$$

By matching E_y and H_z at the appropriate boundaries, one obtains

$$a = \delta_1 + \delta_2 + \frac{1}{k_3}\tan^{-1}\frac{N_1 \sin k_2\delta_2 - N_2 \sin k_1\delta_1}{\sin k_1\delta_1 \sin k_2\delta_2 + N_1N_2} \quad (3)$$

where

$$N_1 = \frac{\mu_0}{k_3}\left[\frac{K_1\gamma}{\mu_1\mu_{e1}} \sin k_1\delta_1 + \frac{k_1}{\mu_{e1}} \cos k_1\delta_1\right]$$

$$N_2 = \frac{\mu_0}{k_3}\left[\frac{K_2\gamma}{\mu_2\mu_{e2}} \sin k_2\delta_2 - \frac{k_2}{\mu_{e2}} \cos k_2\delta_2\right]$$

By substituting (2) into (3) one obtains a transcendental equation for the propagation constant γ. For the particular cases of identical slabs ($\delta_1=\delta_2$; $k_1=k_2$) and a single slab against the side wall ($\delta_2=0$), (3) becomes similar to the results obtained originally by Lax and Button [2].

The magnetic field component H_x can readily be obtained [6] from (1) to give

$$H_x = -\frac{A}{\omega\mu_{e1}}\left[\gamma \sin k_1\left(\frac{a}{2}+x\right) + \frac{K_1k_1}{\mu_1} \cdot \cos k_1\left(\frac{a}{2}+x\right)\right] \quad \text{Region I}$$

$$H_x = -\frac{B}{\omega\mu_{e2}}\left[\gamma \sin k_2\left(\frac{a}{2}-x\right) - \frac{K_2k_2}{\mu_2} \cdot \cos k_2\left(\frac{a}{2}-x\right)\right] \quad \text{Region II}$$

$$H_x = -\frac{\gamma}{\omega\mu_0}\left[C \sin k_3\left(\frac{a}{2}+x\right) + D \cdot \cos k_3\left(\frac{a}{2}+x\right)\right] \quad \text{Region III} \quad (4)$$

where the common factor $e^{j(\omega t-\gamma z)}$ is understood.

From these relationships, derived by matching E_y and H_z at the boundaries, one may also obtain

$$B = \frac{A}{\sin k_2\delta_2}\left[\sin k_1\delta_1 \cos k_3(a-\delta_1-\delta_2) + N_1 \sin k_3(a-\delta_1-\delta_2)\right]$$

$$C = A\left[\sin k_1\delta_1 \sin k_3\delta_1 + N_1 \cos k_3\delta_1\right]$$

$$D = A\left[\sin k_1\delta_1 \cos k_3\delta_1 - N_1 \sin k_3\delta_1\right] \quad (5)$$

In order to calculate the far-zone radiation fields, it is assumed [1] that the field distribution at the open end of the waveguide is the same as if the waveguide were infinite in extent. The far-zone radiation fields will be evaluated from the relationships given by Silver [8]. In the H plane ($\phi=0$) one has

$$E_\phi = \frac{jk}{4\pi R} e^{-jkR} \iint_A (E_y \cos\theta - \eta H_x) \cdot e^{jkx \sin\theta}\, dxdy \quad (6)$$

and in the E plane ($\phi=\pi/2$) one has

$$E_\theta = \frac{jk}{4\pi R} e^{-jkR} \iint_A (E_y - \eta H_x \cos\theta) \cdot e^{jky \sin\theta}\, dxdy \quad (7)$$

where

$$k = \omega\sqrt{\mu_0\epsilon_0} \quad \text{and} \quad \eta = \sqrt{\mu_0/\epsilon_0}$$

The calculation is carried out for each region separately, the final result being the superposition of the three radiation fields. Substituting (1) and (4) into (6), one obtains for the H plane ($\phi=0$) $E_\theta=0$, and E_ϕ for region I is

$$E_\phi = \frac{jAbk}{4\pi} \frac{e^{-jk(R-a/2 \sin\theta)}}{R(k_1^2 - k^2\sin^2\theta)} \cdot \left[k_1\cos\theta + A_1\sin\theta + A_2 + e^{jk\delta_1 \sin\theta}(A_3\sin\theta\cos\theta + A_4\cos\theta + A_5\sin\theta + A_6)\right] \quad (8)$$

where

$$A_1 = -jk\frac{\eta k_1 K_1}{\omega\mu_{e1}\mu_1}; \qquad A_2 = \frac{\eta k_1\gamma}{\omega\mu_{e1}}$$

$$A_3 = jk \sin k_1\delta_1; \qquad A_4 = -k_1\cos k_1\delta_1$$

$$A_5 = \frac{jk\eta}{\omega\mu_{e1}}\left(\gamma \sin k_1\delta_1 + \frac{k_1K_1}{\mu_1}\cos k_1\delta_1\right)$$

$$A_6 = -\frac{\eta k_1}{\omega\mu_{e1}}\left(-\frac{k_1K_1}{\mu_1}\sin k_1\delta_1 + \gamma\cos k_1\delta_1\right)$$

A similar result is obtained for region II, where one should use (8), taking B, k_2, K_2, μ_{e2} instead of A, k_1, K_1, μ_{e1}, and $(-\delta_2)$ instead of δ_1. The results for region III may be found in Chien [9].

Substituting (1) and (4) into (7), one obtains for the E plane ($\phi=\pi/2$)$E_\phi=0$, and E_θ for region I is

$$E_\theta = \frac{A}{2\pi k_1 R}\frac{\sin(\frac{1}{2}bk\sin\theta)}{\sin\theta} \cdot e^{-jkR}\left[D_1\cos\theta + D_2\right] \quad (9)$$

where

$$D_1 = \frac{\eta}{\omega\mu_{e1}}\left[\gamma(1-\cos k_1\delta_1) + \frac{k_1K_1}{\mu_1}\sin k_1\delta_1\right]$$

$$D_2 = 1 - \cos k_1\delta_1$$

A similar result is obtained for region II by use of the transformation already de-

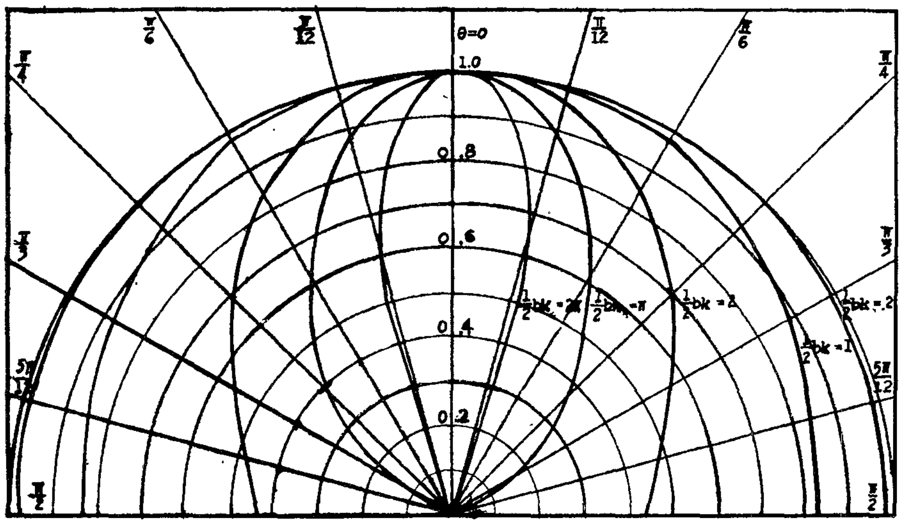

Fig. 2. Electric field pattern E_θ in the E plane ($\phi = \pi/2$) for $\alpha/\beta = 1$.

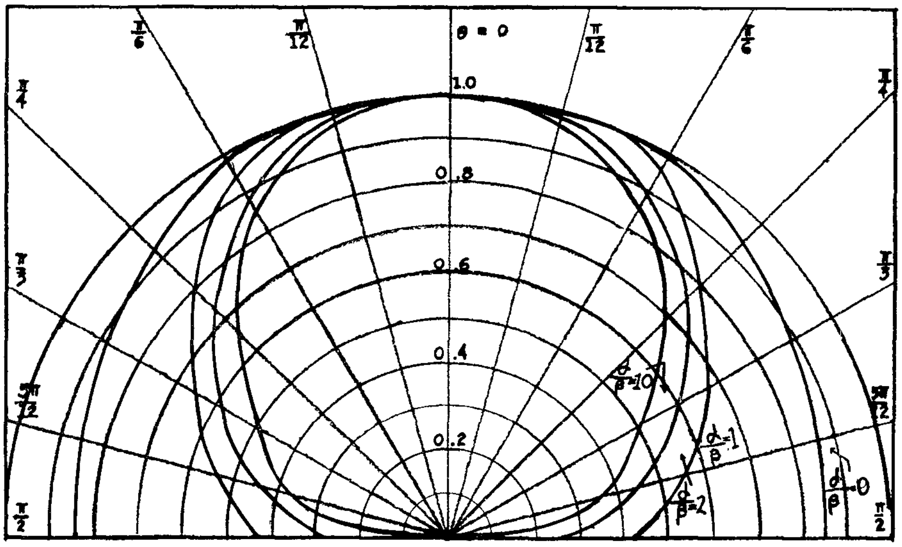

Fig. 3. Electric field pattern E_θ in the E plane ($\phi = \pi/2$) for $(\frac{1}{2})bk = 1$.

scribed. The result for region III has the same general form as shown in (9) for the radiation pattern [9].

The total radiation pattern of E_θ in the E plane ($\phi = \pi/2$) from the three regions is of the general form

$$F(\theta) = \frac{\sin(\frac{1}{2}bk \sin\theta)}{\sin\theta}(\alpha\cos\theta + \beta) \quad (10)$$

Figure 2 shows the corresponding radiation patterns for various values of $(1/2)bk$ for the case $\alpha/\beta = 1$. Figure 3 shows the corresponding radiation patterns for various values of α/β for the case $(1/2)bk = 1$. The radiation patterns are normalized in all cases. The values of α/β depend on the waveguide configuration and the applied static magnetic fields in magnitude and direction.

It has been shown experimentally by Angelakos and Korman [10], and theoretically by Tyras and Held [1], that by changing the static magnetic field in a waveguide filled with ferrite, a scanning of the radiation pattern is obtained in the H plane. A similar scanning should be obtained if one calculates the radiation pattern from (8) for two or more modes of propagation. However, under the present system, more control is given on the radiation pattern from the design point of view (δ_1 *and* δ_2) and from the possibilities of changing the static magnetic fields (H_1 *and* H_2). Additional study of the present system is under way.

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Microwave and High-Frequency Calibration Services of the National Bureau of Standards—Part IV

Introduction

This is the fourth of a series of presentations on microwave and high-frequency calibration services of the National Bureau of Standards which began in the July 1964 issue of these Transactions. (See the July issue for a more complete introduction.) Included in this issue are the services for voltage, frequency stability, and cavity wavemeter calibrations. This completes the listing of presently available microwave and high-frequency calibration services of NBS. It is expected that the announcement of additional calibration services will be published in a similar manner in the Transactions as they become available. As in previous installments, following the listing of calibration services is a series of charts (Figs. 1 and 2) indicating the magnitudes of quantities, the frequency range, and the overall estimated accuracy of the calibrations performed.

Microwave Region

201.930 *Frequency measurements on cavity wavemeters*

Frequency measurements are made on fixed or variable cavity wavemeters of either the reaction (one-port) type or the transmission (two-port) type. Frequency measurements are made on cavity wavemeters

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